

PROBLEME CU ECUATII SI INECUATII (2)
EX. RECAPITULATIVE

26/60. Not.
~~x ∈ Z~~

$$x = n\mathbb{Z}, \quad x \geq 0$$

$$x - 2013 \leq -2011$$

$$x \leq -2011 + 2013$$

$$\left. \begin{array}{l} x \leq 2 \\ x \geq 0 \end{array} \right\} \Rightarrow x \in \{0, 1, 2\}$$

27/60. $(x-1), (x-2), (x-3), \dots, (x-2018) =$ nr. întregi

a) $P = ? \quad x = 2018$

$$P = (2018-1) \cdot (2018-2) \cdot \dots \cdot (2018-2018) =$$

$$= 2017 \cdot 2016 \cdot \dots \cdot 0 = 0$$

b) $P = ? \quad x = 2019$

$$P = (2019-1)(2019-2) \cdot \dots \cdot (2019-2018) = 2018 \cdot 2017 \cdot \dots \cdot 1$$

$$x = 0 \Rightarrow P = (0-1) \cdot (0-2) \cdot \dots \cdot (0-2018) = (-1) \cdot (-2) \cdot \dots \cdot (-2018) =$$

$$\Rightarrow P \text{ este același pt. } x=0 \text{ și } x=2019, \quad = 2018 \cdot 2017 \cdot \dots \cdot 1$$

28/60. Suma ~~to~~ a 2019 nr. întregi consecutive este 0,

$$x + (x+1) + (x+2) + \dots + (x+2018) = 0$$

$$2019 \cdot x + 1 + 2 + 3 + \dots + 2018 = 0$$

$$2019 \cdot x + \frac{2018 \cdot 2019}{2} = 0$$

$$2019 \cdot x + 1009 \cdot 2019 = 0$$

$$2019 \cdot (x + 1009) = 0$$

$$\Rightarrow x + 1009 = 0$$

$$x = -1009$$

elkumerede sunt: $-1009, -1008, \dots, -1, 0, 1, \dots, 1008, 1009$.

$$\begin{aligned} \text{a) } |x| + |x+1| + \dots + |x+2018| &= |1009| + |-1008| + \dots + |0| + |1| + \dots + |2009| \\ &= 1009 + 1008 + \dots + 1 + 0 + 1 + \dots + 1009 = 2 \cdot (1 + 2 + 3 + \dots + 1009) = \\ &= 2 \cdot \frac{1009 \cdot 1010}{2} = 1009 \cdot 1010 \end{aligned}$$

b) $P = ?$

$$P = (-1009) \cdot (-1008) \cdot \dots \cdot 0 \cdot \dots \cdot 1009 = 0$$

TEST 1 (6.3) $x \in \mathbb{Z} = ?$

a) $x^2 - x = 4$

$$\left. \begin{array}{l} x(x-1) = 4 \\ x \in \mathbb{Z} \end{array} \right\} \Rightarrow x \in \emptyset$$

b) $x^3 - x^2 = 100$

$$x^2 \cdot (x-1) = 100 \Rightarrow x=5. \Rightarrow S = \{5\}.$$

$$25 \cdot (5-1) = 100$$

4) $A = \{x \mid x \in \mathbb{Z}, x \leq 3 \text{ or } x > -4\} = \{-3, -2, -1, 0, 1, 2, 3\}$

$$B = \{x \mid x \in \mathbb{Z}, -3 < x \leq 4\} = \{-2, -1, 0, 1, 2, 3, 4\}$$

$$A \cup B = \{-3, -2, -1, 0, 1, 2, 3, 4\}$$

$$A \cap B = \{-2, -1, 0, 1, 2, 3\}$$

$$A \setminus B = \{-3\}$$

T3/61.

$$2. a) A = \{x \in \mathbb{Z} \mid \{13\} / \frac{7}{x-1} \in \mathbb{Z}\}$$

$$\frac{7}{x-1} \in \mathbb{Z} \Rightarrow x-1 \mid 7 \Rightarrow x-1 \in D_7 \Rightarrow x-1 \in \{-7, -1, 1, 7\} / +1$$
$$x \in \{-6, 0, 2, 8\}$$

$$A = \{-6, 0, 2, 8\}$$

$$b) B = \{x \in \mathbb{N} \setminus \{2\} \mid \frac{15}{x-2} \in \mathbb{Z}\}$$

$$\frac{15}{x-2} \in \mathbb{Z} \Rightarrow x-2 \mid 15 \Rightarrow x-2 \in \{1, 3, 5, 15, -1, -3, -5, -15\} / +2$$
$$x \in \{3, 5, 7, 17, 1, -1, -3, -13\}$$

$$B = \{3, 5, 7, 17, 1, -1, -3, -13\}$$

$$c) C = \{x \in \mathbb{Z}^* \mid \frac{12}{x} \in \mathbb{Z} \text{ și } |x| = -x\}$$

$$\frac{12}{x} \in \mathbb{Z} \Rightarrow x \in D_{12} \Rightarrow x \in \{1, 2, 3, 4, 6, 12, -1, -2, -3, -4, -6, -12\}$$
$$|x| = -x$$

$$\Rightarrow C = \{-1, -2, -3, -4, -6, -12\}$$

$$d) D = \{x \in \mathbb{Z} \setminus \{2\} \mid x-1 < 3 \text{ și } \frac{6}{x-2} \in \mathbb{Z}\}$$

$$x-1 < 3 \Rightarrow x < 3+1 \Rightarrow x < 4 \quad (1)$$

$$\frac{6}{x-2} \in \mathbb{Z} \Rightarrow x-2 \in D_6 \Rightarrow x-2 \in \{1, 2, 3, 6, -1, -2, -3, -6\} / +2$$

$$x \in \{3, 4, 5, 8, 1, 0, -1, -4\} \quad (2)$$

$$D \text{ în } (1) \text{ și } (2) \Rightarrow D = \{-4, -1, 0, 1, 3\}$$

TEMA: cul.

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