

NOTIUNI DE TRIGONOMETRIE

Într-un triunghi^{dreptunghic}, relațiile dintre măsurile unghiurilor și lungimile laturilor se numesc relații (funcții) trigonometrice.

Funcțiile trigonometrice sunt: sinus (sin), cosinus (cos), tangenta (tg), cotangenta (ctg).

Pentru un unghi ascuțit^(x) al triunghiului dreptunghic, definiții:

$$\begin{aligned}\sin x &= \frac{\text{cateta opusă}}{\text{ipotenuză}} = \frac{\text{cat. op.}}{\text{ip.}} \\ \cos x &= \frac{\text{cateta alăturată}}{\text{ipotenuză}} = \frac{\text{cat. al.}}{\text{ip.}} \\ \text{tg } x &= \frac{\sin x}{\cos x} = \frac{\text{cat. op.}}{\text{cat. al.}} \\ \text{ctg } x &= \frac{1}{\text{tg } x} = \frac{\cos x}{\sin x} = \frac{\text{cat. al.}}{\text{cat. op.}}\end{aligned}$$

În orice triunghi dreptunghic, $\sin^2 x + \cos^2 x = 1$

Pentru unghiurile complementare (cu suma 90°):

$$\begin{aligned}\sin(90^\circ - x) &= \cos x & \text{tg}(90^\circ - x) &= \text{ctg } x \\ \cos(90^\circ - x) &= \sin x & \text{ctg}(90^\circ - x) &= \text{tg } x\end{aligned}$$

Valorile funcțiilor trigonometrice pentru unghiurile de 30° , 45° , 60° sunt:

	sin	cos	tg	ctg
30°	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$

TEOREMA COSINUSULUI

În triunghiul oarecare ABC , cu $m(\hat{C}) < 90^\circ$; $D = p_{BC}^A$

$$AB^2 = BC^2 + AC^2 - 2 \cdot BC \cdot AC \cdot \cos C$$

În Δ oarecare ABC , cu $m(\hat{C}) > 90^\circ$, $D = p_{BC}^A$

$$AB^2 = BC^2 + AC^2 + 2 \cdot BC \cdot AC \cdot \cos(180^\circ - C)$$

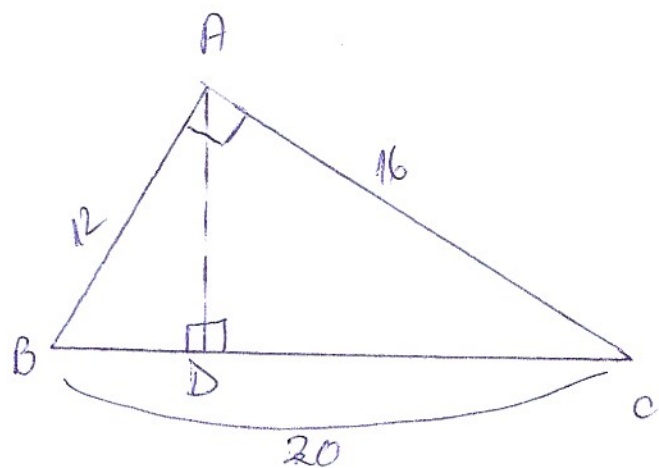
EXERCITII:

ΔABC , $m(\hat{A}) = 90^\circ$

a) $AB = 12 \text{ cm}$

$AC = 16 \text{ cm}$

Cl: BC , $\sin B$, $\sin C$, $\cos B$,
 $\cos C$, $\text{tg } B$, $\text{tg } C$, $\text{ctg } B$, $\text{ctg } C$



Dem:

În ΔABC , $m(\hat{A}) = 90^\circ \xRightarrow{\text{v.p.}}$ $BC^2 = AB^2 + AC^2$
 $BC^2 = 12^2 + 16^2$

$$BC^2 = 4^2 \cdot (3^2 + 4^2)$$

$$BC^2 = 4^2 \cdot 5^2 \Rightarrow BC = \sqrt{4^2 \cdot 5^2} = 4 \cdot 5 = 20 \text{ cm}$$

$$\sin B = \frac{\text{cat. op}}{\text{ip}} = \frac{AC}{BC} = \frac{16}{20} = \frac{4}{5}$$

$$\cos B = \frac{\text{cat. ad}}{\text{ip}} = \frac{AB}{BC} = \frac{12}{20} = \frac{3}{5}$$

$$\sin C = \frac{AB}{BC} = \frac{12}{20} = \frac{3}{5}$$

$$\cos C = \frac{AC}{BC} = \frac{16}{20} = \frac{4}{5}$$

$$\text{tg } B = \frac{\text{cat. op}}{\text{cat. ad}} = \frac{AC}{AB} = \frac{16}{12} = \frac{4}{3}$$

$$\text{ctg } B = \frac{\text{c. ad}}{\text{c. op}} = \frac{AB}{AC} = \frac{12}{16} = \frac{3}{4}$$

$$\text{ctg } B = \frac{\text{cat. ad}}{\text{cat. op}} = \frac{AB}{AC} = \frac{12}{16} = \frac{3}{4}$$

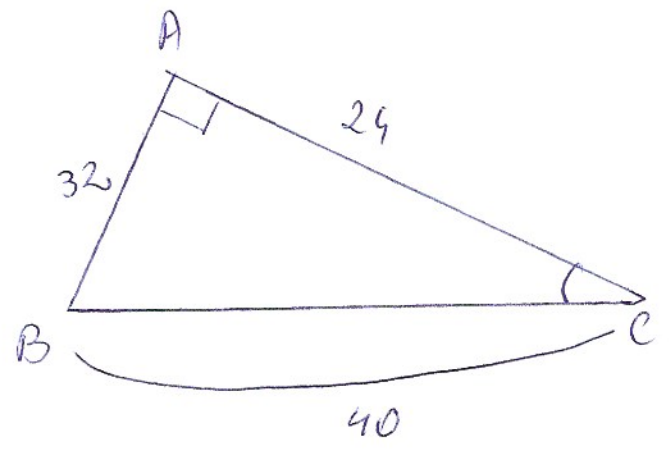
$$\text{ctg } C = \frac{AC}{AB} = \frac{16}{12} = \frac{4}{3}$$

3/84. $\hat{A} = 90^\circ$

c) $BC = 40 \text{ cm}$

$$\sin C = \frac{4}{5}$$

el: $AB, AC, \sin B,$
 $\cos B, \cos C, \text{tg } C, \text{ctg } C.$



Dem:

$$\text{In } \triangle ABC, \hat{A} = 90^\circ \Rightarrow \sin C = \frac{\text{cat. op}}{\text{ip}} \Rightarrow \sin C = \frac{AB}{BC}$$

$$\frac{4}{5} = \frac{AB}{40} \Rightarrow$$

$$\Rightarrow AB = \frac{4 \cdot 40}{5} = \boxed{32 \text{ cm}}$$

T.P. $\Rightarrow AC^2 = BC^2 - AB^2$

$$AC^2 = 40^2 - 32^2 \Rightarrow AC^2 = 8^2 \cdot (5^2 - 4^2) = 8^2 \cdot 3^2 \Rightarrow AC = 8 \cdot 3 = \boxed{24 \text{ cm}}$$

$$\sin B = \frac{AC}{BC} = \frac{24}{40} = \frac{3}{5}$$

$$\text{tg } C = \frac{AB}{AC} = \frac{32}{24} = \frac{4}{3}$$

$$\cos C = \frac{AC}{BC} = \frac{24}{40} = \frac{3}{5}$$

$$\text{ctg } C = \frac{AC}{AB} = \frac{24}{32} = \frac{3}{4}$$

$$\cos B = \frac{AB}{BC} = \frac{32}{40} = \frac{4}{5}$$

5. \sqrt{f} : ABCD - trapez dr.

$AD \parallel BC$

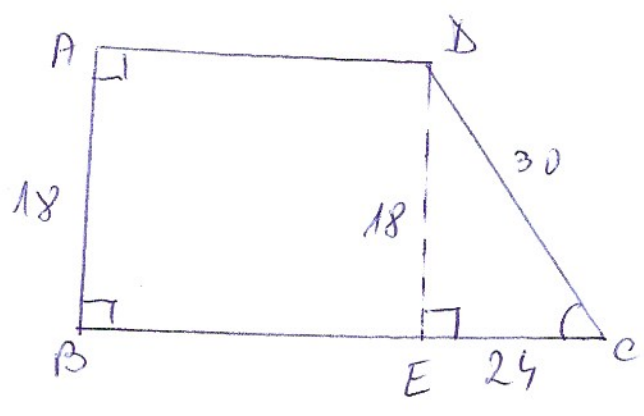
$AD < BC$

$m(\hat{A}) = m(\hat{B}) = 90^\circ$

$h = 18 \text{ cm}$

$\text{tg } C = 0,75$

cl: DC = ?



Dem:

Fie $DE \perp BC \Rightarrow ADEB$ - dreptunghi \Rightarrow

$\Rightarrow DE = AB = 18 \text{ cm}$

In $\triangle DEC$, $m(\hat{E}) = 90^\circ \Rightarrow \text{tg } C = \frac{DE}{EC}$

$0,75 = \frac{DE}{EC}$

$$\frac{75}{100} = \frac{18}{EC} \Rightarrow EC = \frac{100 \cdot 18}{75}$$

$$EC = 24 \text{ cm}$$

T.P. $\Rightarrow DC^2 = DE^2 + EC^2$

$DC^2 = 18^2 + 24^2$

$DC^2 = 6^2 \cdot (3^2 + 4^2)$

$DC^2 = 6^2 \cdot 5^2 \Rightarrow DC = \sqrt{6^2 \cdot 5^2} = 6 \cdot 5 = \underline{30 \text{ cm}}$

TEMA: cul. pag 84 / Ex: 1b, 2d, 3d, 6, 7.