LECȚIA 4 (1) 28.04.

GEOMETRIE

CLASELE VI B, D

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PROBLEME – PROPRIETĂȚILE TRIUNGHIULUI ISOSCEL

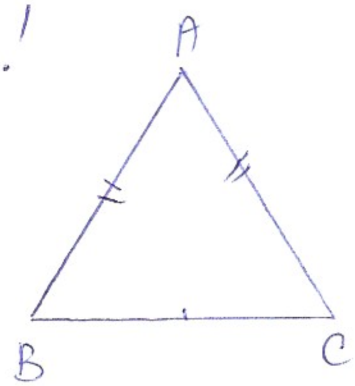
13/166 culegere: Demonstrați că în orice triunghi isoscel:

a) unghiurile de la baza triunghiului sunt congruente

b) bisectoarele unghiurilor de la bază sunt congruente

c) medianele corespunzătoare laturilor congruente sunt congruente;

d) înălțimile corespunzătoare laturilor congruente sunt congruente. DE REȚINUT!



Demonstrație:

a) ΔABC – isoscel

$[AB] \equiv [AC]$

$\measuredangle B \equiv \measuredangle C$

Dem:

Din $\left.\begin{array}{l}[AB] \equiv [AC] \\ [AC] \equiv [AB] \\ [BC] \equiv [BC]\ (l.c.)\end{array}\right\} \overset{L.L.L}{\Rightarrow} \Delta ABC \equiv \Delta ACB$

$\Downarrow$

$\measuredangle B \equiv \measuredangle C$

b) ΔABC – is.

$[AB] \equiv [AC]$

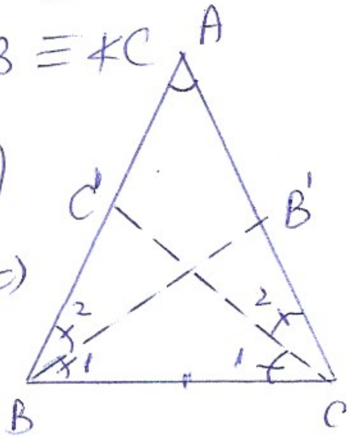
$[BB'$ – bis $\measuredangle B$

$[CC'$ – bis $\measuredangle C$

Cl: $[BB'] \equiv [CC']$

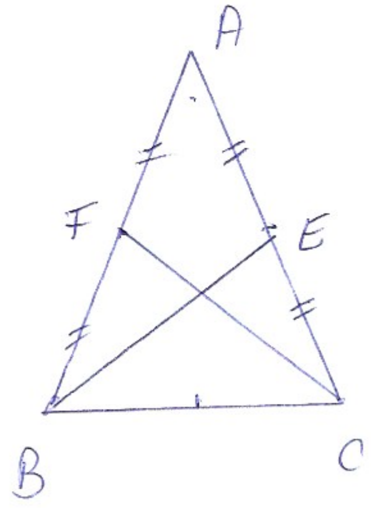
Dem:

$\left.\begin{array}{l}\text{Din } \Delta ABC\text{ – is} \Rightarrow \measuredangle B \equiv \measuredangle C \\ [BB'\text{ – bis } \measuredangle B \Rightarrow \measuredangle B_1 \equiv \hat{B}_2 \\ [CC'\text{ – bis } \measuredangle C \Rightarrow \hat{C}_1 \equiv \hat{C}_2\end{array}\right\} \Rightarrow \hat{B}_1 \equiv \hat{B}_2 \equiv \hat{C}_1 \equiv \hat{C}_2$



Fie $\Delta ABB'$ și $\Delta ACC'$

Din $\left.\begin{array}{l}[AB] \equiv [AC] \\ \sphericalangle A \equiv \sphericalangle A \\ \sphericalangle B_2 \equiv \sphericalangle C_2\end{array}\right\} \xRightarrow{ULU} \Delta ABB' \equiv \Delta ACC' \Rightarrow [BB'] \equiv [CC']$



c) ΔABC - is.

$[AB] \equiv [AC]$

BE, CF - mediane

$E \in (AC)$

$F \in (AB)$

Cl: $[BE] \equiv [CF]$.

Dem:

$\left.\begin{array}{l}\text{Din } BE\text{- mediană} \Rightarrow E = \text{mijl}(AC) \Rightarrow AE = EC = \frac{AC}{2} \\ \text{Din } CF\text{- mediană} \Rightarrow F\text{- mijl}(AB) \Rightarrow AF = FB = \frac{AB}{2} \\ [AC] \equiv [AB]\end{array}\right\} \Rightarrow$

$\Rightarrow AF = FB = AE = EC$

Fie ΔAEB și ΔAFC

Din $\left.\begin{array}{l}[AE] \equiv [AF] \\ [AB] \equiv [AC] \\ \sphericalangle A \equiv \sphericalangle A\end{array}\right\} \xRightarrow{LUL} \Delta AEB \equiv \Delta AFC$

$\Downarrow$

$[BE] \equiv [CF]$

d) ΔABC - is.

$[AB] \equiv [AC]$

$BE \perp AC$, $E \in (AC)$

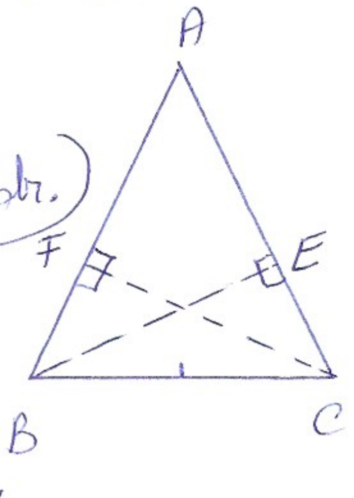
$CF \perp AB$, $F \in (AB)$

Cl: $[BE] \equiv [CF]$.

Dem:

Fie ΔAEB și ΔAFC - dr.

$\left.\begin{array}{l}AB \equiv AC \\ \sphericalangle A \equiv \sphericalangle A\end{array}\right\} \Rightarrow$



$\xRightarrow{i.U} \Delta AEB \equiv \Delta AFC \Rightarrow [BE] \equiv [CF]$.

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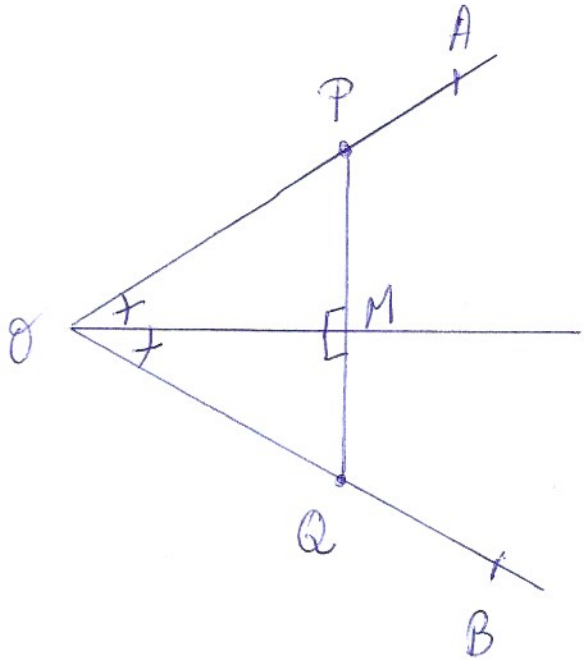

Ip:

$\sphericalangle AOB$

$M \in$ bis. $\sphericalangle AOB$

$\perp$ în M pe $OM \cap [OA = \{P\}$

$\cap [OB = \{Q\}$

Cl: $OP \equiv OQ$

Dem:

Din $OM \perp PQ \Rightarrow OM =$ înălțime în ΔOQP
Din $OM -$ bis. $\sphericalangle O$
$\Big\} \xRightarrow{\text{Propr. 4}} \Delta OPQ -$ is.
$\Downarrow$
$[OP] \equiv [OQ]$

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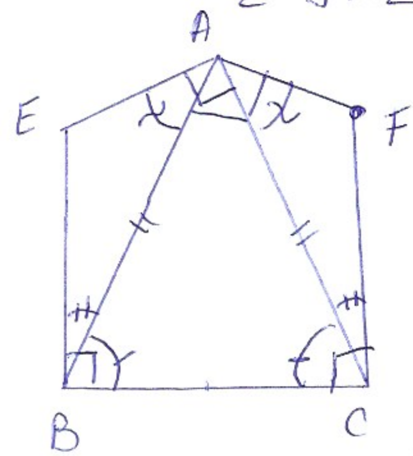

Ip:

$\Delta ABC -$ is.

$BC =$ bază

$\perp$ în A pe $AC \cap \perp$ în B pe $BC = \{E\}$

$\perp$ în A pe $AB \cap \perp$ în C pe $BC = \{F\}$

Cl: $BE \equiv CF$

Dem:

Din $\Delta ABC -$ is.
$BC =$ bază
$\Big\} \Rightarrow [AB] \equiv [AC]$

$\sphericalangle B \equiv \sphericalangle C$
$m(\widehat{EBA}) + m(\widehat{ABC}) = 90^\circ$
$m(\widehat{FCA}) + m(\widehat{ACB}) = 90^\circ$
$\Big\} \Rightarrow \sphericalangle EBA \equiv \sphericalangle FCA$

$m(\widehat{EAB}) + m(\widehat{BAC}) = 90^\circ$
$m(\widehat{FAC}) + m(\widehat{BAC}) = 90^\circ$
$\Big\} \Rightarrow \sphericalangle EAB \equiv \sphericalangle FAC$

$AB \equiv AC$

$\xRightarrow{ULU} \Delta EAB \equiv$
$\equiv \Delta FAC$
$\Downarrow$
$EB \equiv CF$

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Ip:

ΔABC - is.

$[AB] \equiv [AC]$

$D \in (AB)$

$E \in (AC)$

$BD \equiv CE$

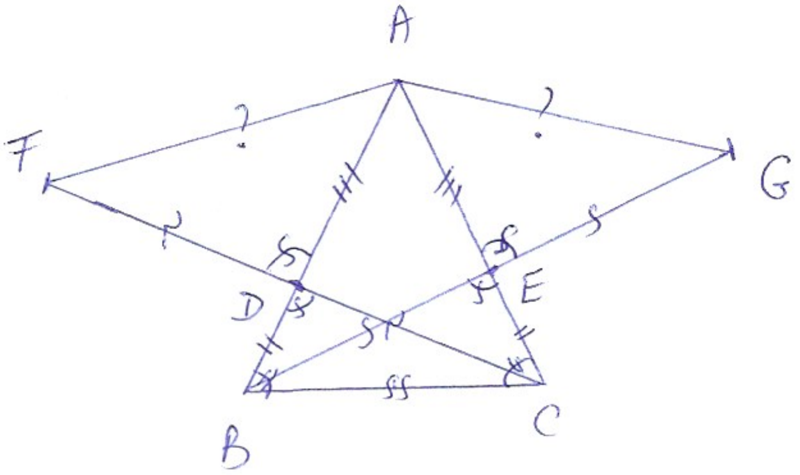
$F \in [CD$

$G \in [BE$

$CF = 2 \cdot CD$

$BG = 2 \cdot BE$

Cl: $AF \equiv AG$



Dem:

Din $\left.\begin{array}{l}\Delta ABC \text{ - is.} \\ [AB] \equiv [AC]\end{array}\right\} \Rightarrow \angle B \equiv \angle C$

Fie ΔDBC și ΔECB

Din $\left.\begin{array}{l} DB \equiv EC \\ \angle B \equiv \angle C \\ BC \equiv BC \text{ (lat. com.)}\end{array}\right\} \xrightarrow{LUL} \Delta DBC \equiv \Delta ECB$

$\Downarrow$

$\left.\begin{array}{l} DC \equiv BE \\ \text{Dar } DC \equiv FD \\ BE \equiv GE \end{array}\right\} \Rightarrow$

$\Rightarrow FD \equiv GE$

Tot din $\Delta DBC \equiv \Delta ECB \Rightarrow \angle BDC \equiv \angle CEB \Rightarrow \angle ADF \equiv \angle AEG$

$\left.\begin{array}{l} AD = AB - BD \\ AE = AC - EC \\ BD \equiv EC \end{array}\right\} \Rightarrow AD \equiv AE$

Fie ΔADF și ΔAEG

Din $\left.\begin{array}{l} AD \equiv AE \\ \angle ADF \equiv \angle AEG \\ FD \equiv EG \end{array}\right\} \xrightarrow{L.U.L.} \Delta ADF \equiv \Delta AEG$

$\Downarrow$

$AF \equiv AG$.

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