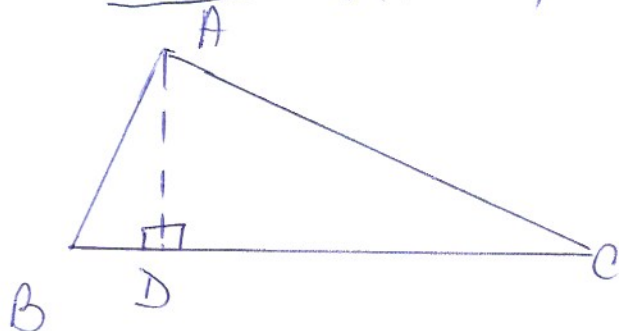


PROBLEME - APROFUNDARE TEOREMAÎNĂLȚIMII ȘI TEOREMA CATETEI17/63 TEORIE: În  $\triangle ABC$ ,  $m(\hat{A}) = 90^\circ \xRightarrow{T.C.}$   $AD^2 = BD \cdot DC$ 

$$\xRightarrow{T.C.} AB^2 = BD \cdot BC$$

$$AC^2 = CD \cdot CB$$



17/63 culegere

I.P.:  $\triangle ABC$ ,  $m(\hat{A}) = 90^\circ$ 

$AB < AC$

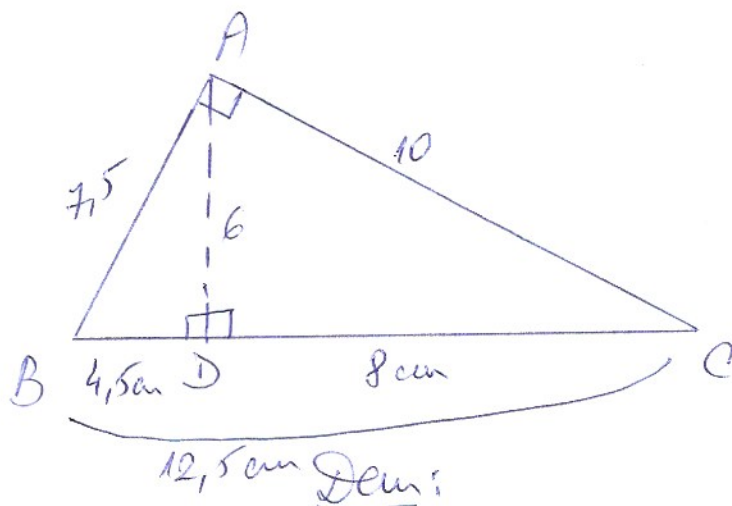
$AD \perp BC$

$D \in (BC)$

a)  $4BD - CD = 12 \text{ cm}$

$$\frac{CD}{BD} = 1 \frac{7}{9}$$

C.:  $BD, CD, BC, AB, AC, AD = ?$

12,5 cm Dem:

$$\frac{CD}{BD} = 1 \frac{7}{9} \Rightarrow \frac{CD}{BD} = \frac{16}{9} \Rightarrow$$

$$\Rightarrow CD = 16K$$

$$BD = 9K, K \in \mathbb{N}^*$$

Înlocuim în  $4BD - CD = 12 \text{ cm} \Rightarrow 4 \cdot 9K - 16K = 12$

$$24K = 12 \quad |:24$$

$$K = \frac{1}{2} \Rightarrow CD = 16 \cdot \frac{1}{2} = 8 \text{ cm}$$

$$\Rightarrow BC = BD + DC = 12,5 \text{ cm}$$

În  $\triangle ABC$ ,  $m(\hat{A}) = 90^\circ \xRightarrow{T.C.} AB^2 = BD \cdot BC$

$$BD = 9 \cdot \frac{1}{2} = 4,5 \text{ cm}$$

$$AB^2 = 4,5 \cdot 12,5 = \frac{45}{10} \cdot \frac{125}{10} = \frac{9 \cdot 5 \cdot 25 \cdot 5}{10^2}$$

$$AB = \sqrt{\frac{9 \cdot 25 \cdot 25}{100}} = \frac{3 \cdot 5 \cdot 5}{10} = \frac{75}{10} = 7,5 \text{ cm}$$

- (2) -

$$\text{T.C.} \Rightarrow AC^2 = CD \cdot CB$$

$$AC^2 = 8 \cdot 12,5 = \frac{4}{10} \cdot \frac{25}{51} = 100 \Rightarrow AC = \sqrt{100} = 10 \text{ cm}$$

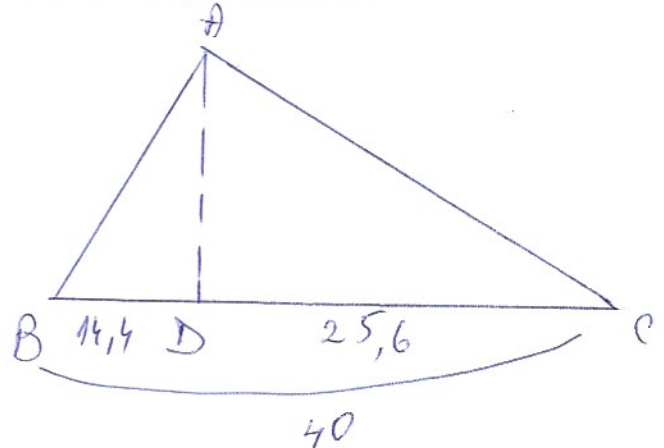
$$\text{T.I.} \Rightarrow AD^2 = BD \cdot DC$$

$$AD^2 = 4,5 \cdot 8 = 36,0 \Rightarrow AD = \sqrt{36} = 6 \text{ cm}$$

$$b) CD - BD = \frac{56}{5} \text{ cm}$$

$$BC = 40 \text{ cm}$$

$$\underline{Q} : BD, CD, AB, AC, AD = ?$$



Dem :

$$CD + BD = BC \Rightarrow CD + BD = 40$$

$$CD - BD = 11,2 \quad (+)$$

$$\hline 2CD = 51,2$$

$$CD = 51,2 : 2$$

$$\boxed{CD = 25,6 \text{ cm}} \Rightarrow BD = 40,0 - 25,6$$

$$\boxed{BD = 14,4 \text{ cm}}$$

$$\text{In } \triangle ABC, m(\hat{A}) = 90^\circ \xRightarrow{\text{T.C.}}$$

$$AB^2 = BD \cdot BC$$

$$AB^2 = 14,4 \cdot 40 = \frac{144}{10} \cdot 40 \Rightarrow AB = 12 \cdot 2$$

$$\boxed{AB = 24 \text{ cm}}$$

$$AC^2 = CD \cdot CB$$

$$AC^2 = 25,6 \cdot 40 = \frac{256}{10} \cdot 40 \Rightarrow AC = 16 \cdot 2$$

$$\boxed{AC = 32 \text{ cm}}$$

$$\text{T.I.} \Rightarrow AD^2 = BD \cdot DC$$

$$AD^2 = 14,4 \cdot 25,6 = \frac{144}{10} \cdot \frac{256}{10} \Rightarrow AD = \frac{12 \cdot 16}{10}$$

$$AD = \frac{192}{10} \Rightarrow \boxed{AD = 19,2 \text{ cm}}$$

③-

23/63.  $\triangle ABC$

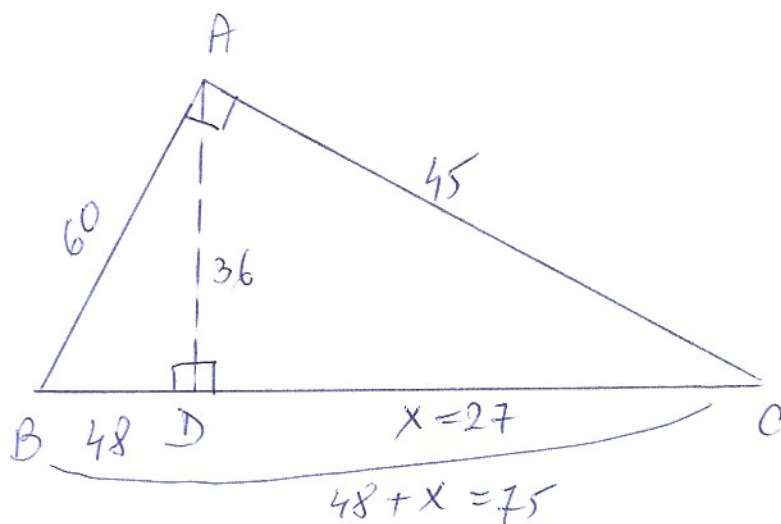
$$m(\hat{A}) = 90^\circ$$

$$AD \perp BC$$

$$D \in (BC)$$

$$AC = 45 \text{ cm}$$

$$BD = 48 \text{ cm}$$



cl : a)  $AB, AD = ?$

b)  $P_{\triangle ABC} = ?$

c)  $\frac{A_{ACD}}{A_{ABD}} = ?$

Dem:

$$\text{Notăm } CD = x \Rightarrow BC = 48 + x$$

$$\text{În } \triangle ABC, m(\hat{A}) = 90^\circ \xrightarrow{\text{T.C.}} AC^2 = CD \cdot CB$$

$$45^2 = x \cdot (48 + x)$$

$$45 \cdot 45 = x \cdot (48 + x)$$

$$9 \cdot 5 \cdot 3 \cdot 15 = x \cdot (48 + x)$$

$$27 \cdot 75 = x \cdot (48 + x)$$

$$\Rightarrow \boxed{x = 27 \text{ cm}} \Rightarrow \boxed{BC = 48 + 27}$$

$$\boxed{BC = 75 \text{ cm}}$$

$$\xrightarrow{\text{T.C.}} AB^2 = BD \cdot BC$$

$$AB = \sqrt{48 \cdot 75} = \sqrt{16 \cdot 3 \cdot 25 \cdot 3} = 4 \cdot 5 \cdot 3 = 60 \text{ cm}$$

$$\xrightarrow{\text{T.P.}} AD^2 = BD \cdot DC$$

$$AD = \sqrt{48 \cdot 27} = \sqrt{16 \cdot 3 \cdot 9 \cdot 3} = 4 \cdot 3 \cdot 3 = 36 \text{ cm}$$

$$b) P_{\triangle ABC} = 60 + 75 + 45 = 180 \text{ cm}$$

$$A_{\triangle ABC} = \frac{c_1 \cdot c_2}{2} = \frac{60 \cdot 45}{2} = 1350 \text{ cm}^2$$

$$c) \left. \begin{aligned} A_{\triangle ACD} &= \frac{AD \cdot DC}{2} = \frac{36 \cdot 27}{2} = 18 \cdot 27 \\ A_{\triangle ABD} &= \frac{AD \cdot BD}{2} = \frac{36 \cdot 48}{2} = 18 \cdot 48 \end{aligned} \right\} \Rightarrow \frac{A_{ACD}}{A_{ABD}} = \frac{18 \cdot 27}{18 \cdot 48} = \frac{9}{16}$$



27/64  $\frac{7}{p}$

ABCD - trapezoid.

$AB \parallel CD$

$AB > CD$

$AD = DC = BC$

$AC \perp BC$

$EF = l.m.$

$EF \cap AC = \{M\}$

$EF \cap BD = \{N\}$

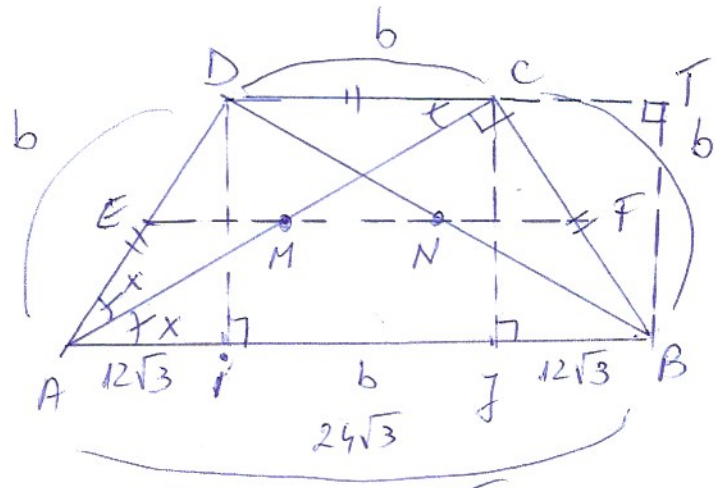
$MN = 12\sqrt{3} \text{ cm}$

a)  $AB, CD = ?$

b)  $AC, h = ?$

c)  $P, \angle ABC = ?$

d)  $\angle ADB = ?$



Dem:  $48\sqrt{3}$

$$EF = l.m. \Rightarrow EF = \frac{B+b}{2} ; AB = B - \text{baza mare}$$

$$MN = \frac{B-b}{2} \quad CD = b = \text{baza mic}$$

$$12\sqrt{3} = \frac{B-b}{2} \Rightarrow B-b = 24\sqrt{3} \text{ cm}$$

$$\Rightarrow AI = JB = \frac{B-b}{2} = \frac{24\sqrt{3}}{2} = 12\sqrt{3} \text{ cm}$$

$\text{Sim } AD = DC \Rightarrow \triangle ADC = \text{is.} \quad \angle DAC = \angle DCA$   
 $\text{Sim } DC \parallel AB \quad AC = \text{secante} \Rightarrow \angle DCA = \angle CAB \text{ (alt. int.)}$

$$\Rightarrow \angle DAC = \angle CAB \text{ not.} \Rightarrow m(\angle DAC) = m(\angle CAB) = x^\circ \Rightarrow m(\angle A) = 2x^\circ$$

$$\Rightarrow \text{In } \triangle ACB, \quad 2x^\circ + 2x^\circ = 90^\circ \Rightarrow 3 \cdot x^\circ = 90^\circ \Rightarrow x^\circ = \frac{90^\circ}{3} = 30^\circ$$

$$\Rightarrow m(\angle A) = 30^\circ \cdot 2 = 60^\circ$$

$$m(\angle B) = 60^\circ$$

$$m(\angle CAB) = 30^\circ$$

$$\left. \begin{array}{l} m(\angle B) = 60^\circ \\ m(\angle CAB) = 30^\circ \end{array} \right\} \text{T. } 30-60-90 \Rightarrow$$

$$CB = \frac{AB}{2} \Rightarrow AB = 2 \cdot CB$$

$$24\sqrt{3} + b = 2b \quad / -b$$

$$24\sqrt{3} = b \Rightarrow B = 24\sqrt{3} + 24\sqrt{3}$$

$$B = 48\sqrt{3} \text{ cm}$$

$$CD = 24\sqrt{3} \text{ cm}$$

$$AB = 48\sqrt{3} \text{ cm}$$

b) In  $\triangle ACB$ ,  $m(\hat{C}) = 90^\circ$  T.C  $AC^2 = AJ \cdot AB$

$$AC = \sqrt{36\sqrt{3} \cdot 48\sqrt{3}} = \sqrt{36 \cdot 3 \cdot 16 \cdot 3}$$

$$AC = 6 \cdot 3 \cdot 4 = 24 \cdot 3 = \boxed{72 \text{ cm}}$$

T.P.  
 $\Rightarrow CJ^2 = AJ \cdot JB$

$$CJ = \sqrt{36\sqrt{3} \cdot 12\sqrt{3}} = \sqrt{36 \cdot 3 \cdot 4 \cdot 3} = 6 \cdot 3 \cdot 2$$

$$\boxed{CJ = 36 \text{ cm}} \Rightarrow \boxed{h = 36 \text{ cm}}$$

c)  $P_{ABCD} = 48\sqrt{3} + 3 \cdot 24\sqrt{3} = 48\sqrt{3} + 72\sqrt{3} = \boxed{120\sqrt{3} \text{ cm}}$

$$A_{ABCD} = \frac{(B+b) \cdot h}{2} = \frac{(48\sqrt{3} + 24\sqrt{3}) \cdot 36}{2} = 72\sqrt{3} \cdot 18 = \boxed{1296\sqrt{3} \text{ cm}^2}$$

d)  $A_{\triangle BCD} = \frac{b \cdot h}{2} = \frac{CD \cdot BT}{2} = \frac{24\sqrt{3} \cdot h}{2} = \frac{24\sqrt{3} \cdot 36}{2} = \boxed{432\sqrt{3} \text{ cm}^2}$

✓  
 $\triangle$  obțuzunghi (baza = DC)  
 $h = BT$

TEMA - până joi, 16, 21 / 63 culegere.