

TEOREMA LUI PITAGORA

TEOREMA: (lui Pitagora): Într-un triunghi dreptunghic, suma pătratelor lungimilor catetelor este egală cu pătratul lungimii ipotenusei.

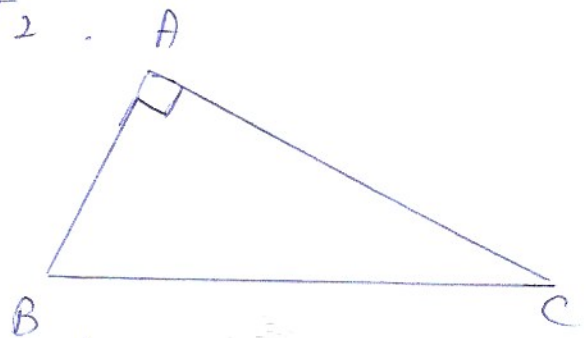
Se not: T.P: $cat_1^2 + cat_2^2 = ip^2$

sau $cat_1^2 = ip^2 - cat_2^2$

Ex: Dacă $\triangle ABC$, $m(\hat{A}) = 90^\circ \stackrel{T.P.}{\Rightarrow}$

$$BC^2 = AB^2 + AC^2$$

sau $AB^2 = BC^2 - AC^2$



RECIPROCA TEOREMEI LUI PITAGORA

Dacă într-un triunghi, suma pătratelor lungimilor a două laturi este egală cu pătratul lungimii laturii celei mai mari, atunci triunghiul este dreptunghic.

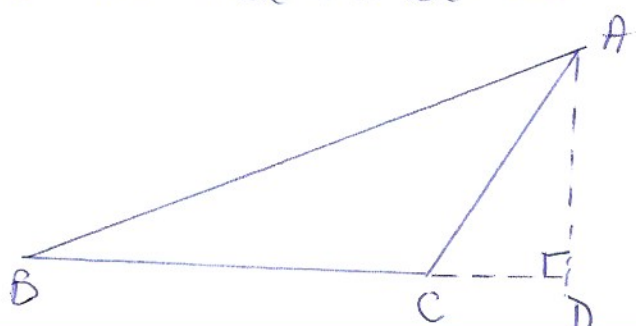
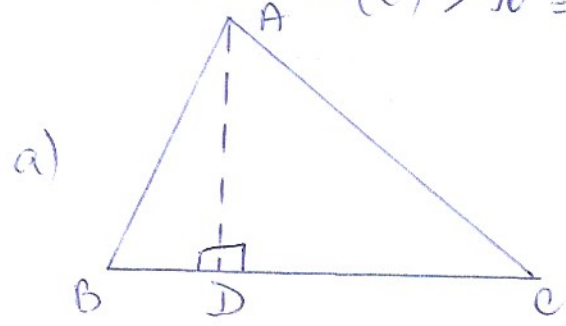
Dacă $AB^2 + AC^2 = BC^2$, atunci $\triangle ABC$ dreptunghic cu $m(\hat{A}) = 90^\circ$

TEOREMA LUI PITAGORA GENERALIZATĂ (extindere)

Fie $\triangle ABC$ și $D = pr_{BC}^A$ ($AD \perp BC$, $D \in BC$).

a) Dacă $m(\hat{C}) < 90^\circ \Rightarrow AB^2 = AC^2 + BC^2 - 2 \cdot BC \cdot CD$

b) Dacă $m(\hat{C}) > 90^\circ \Rightarrow AB^2 = AC^2 + BC^2 + 2 \cdot BC \cdot CD$



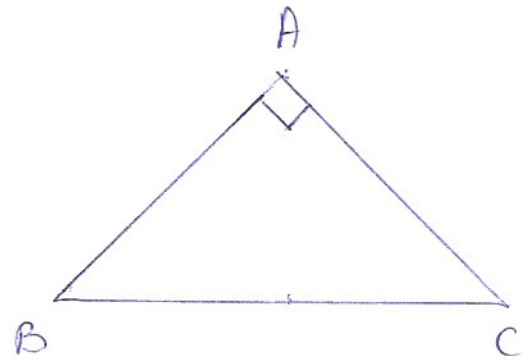
EXERCITIUL:

1/70: I.P.: $\triangle ABC$, $m(\hat{A}) = 90^\circ$

$[AB] \equiv [AC]$

a) $BC = 24\sqrt{6}$ cm

$AB, AC = ?$



Dem:

In $\triangle ABC$, $m(\hat{A}) = 90^\circ \xrightarrow{\text{T.P.}}$ $BC^2 = AB^2 + AC^2$

$(24\sqrt{6})^2 = AB^2 + AB^2$

$24^2 \cdot 6 = 2 \cdot AB^2$

$AB^2 = \frac{24^2 \cdot 6}{2}$

$AB = \sqrt{24^2 \cdot 3} = 24\sqrt{3}$ cm = AC.

b) $AB = 18\sqrt{2} \Rightarrow AC = 18\sqrt{2}$

In $\triangle ABC$, $m(\hat{A}) = 90^\circ \xrightarrow{\text{T.P.}}$ $BC^2 = AB^2 + AC^2$

$BC^2 = (18\sqrt{2})^2 + (18\sqrt{2})^2$

$BC^2 = 18^2 \cdot 2 + 18^2 \cdot 2$

$BC^2 = 18^2 \cdot 4$

$BC = \sqrt{18^2 \cdot 4} = 18 \cdot 2 = 36$ cm

2/70: I.P.:
 $\triangle ABC$

$m(\hat{A}) = 90^\circ$

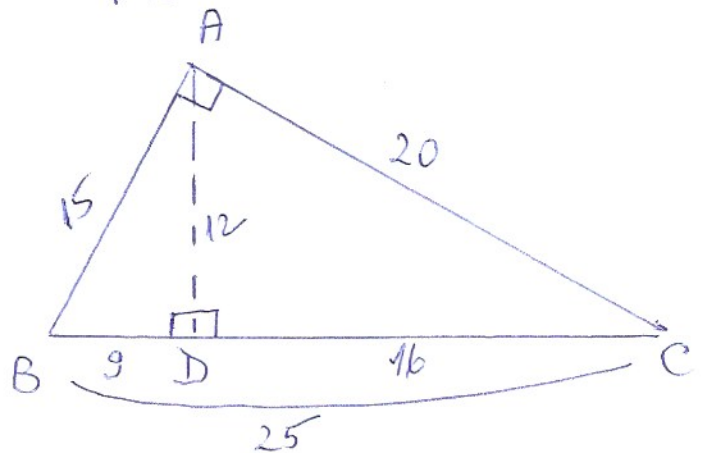
$AD \perp BC$

$DE \perp (BC)$

a) $AB = 15$ cm

$AC = 20$ cm

$BC, CD, AD = ?$



Dem:

In $\triangle ABC$, $m(\hat{A}) = 90^\circ \xrightarrow{\text{T.P.}}$ $BC^2 = AB^2 + AC^2$

$BC^2 = 15^2 + 20^2$

$BC^2 = 5^2 \cdot (3^2 + 4^2)$

-3-

$$BC^2 = 5^2 \cdot 5^2$$

$$BC = \sqrt{5^2 \cdot 5^2} = 5 \cdot 5 = 25 \text{ cm}$$

$$AD = h_{\Delta ABC} = \frac{\text{cat}_1 \cdot \text{cat}_2}{ip}$$

$$AD = \frac{AB \cdot AC}{BC} = \frac{15 \cdot 20}{25} = 12 \text{ cm}$$

NE REAMINTIM :

$$3^2 + 4^2 = 5^2 \Rightarrow 3, 4, 5 - \text{nr.}$$

$$3^2 = 5^2 - 4^2 \quad \text{Pitagoreice}$$

$$4^2 = 5^2 - 3^2$$

$$3, 4, 5 / \cdot 2 \Rightarrow 6, 8, 10 - \text{nr.}$$

Pitagoreice

In ΔADB , $m(\hat{D}) = 90^\circ \xrightarrow{\text{T.P.}}$ $BD^2 = AB^2 - AD^2$

$$BD^2 = 15^2 - 12^2$$

$$BD^2 = 3^2 \cdot (5^2 - 4^2)$$

$$BD^2 = 3^2 \cdot 3^2$$

$$BD = \sqrt{3^2 \cdot 3^2} = 3 \cdot 3 = 9 \text{ cm}$$

$$DC = BC - BD = 25 - 9 = 16 \text{ cm}$$

5/70. I_P :

ΔABC

$$m(\hat{A}) = 90^\circ$$

$AD \perp BC$

$D \in (BC)$

a) $BD = 27 \text{ cm}$

$$AB = 45 \text{ cm}$$

$$CD, AD, AC, BC = ?$$

Dem:

In ΔADB , $m(\hat{D}) = 90^\circ \xrightarrow{\text{T.P.}}$ $AD^2 = AB^2 - BD^2$

$$AD^2 = 45^2 - 27^2$$

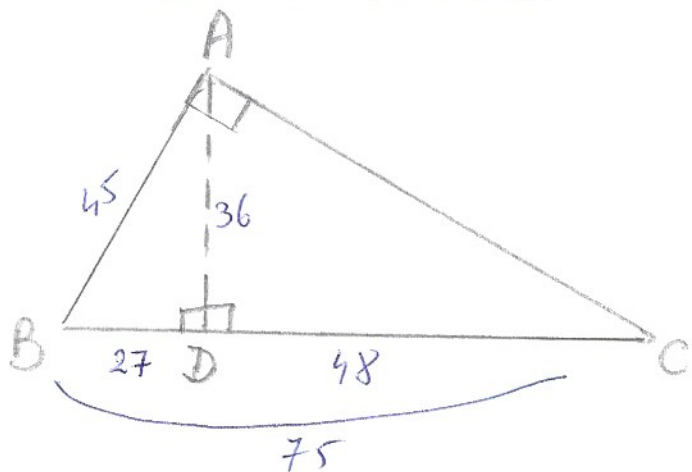
$$AD^2 = 9^2 \cdot (5^2 - 3^2)$$

$$AD^2 = 9^2 \cdot 4^2 \Rightarrow AD = \sqrt{9^2 \cdot 4^2} = 9 \cdot 4 = 36 \text{ cm}$$

T.C $\Rightarrow AB^2 = BD \cdot BC$

$$45^2 = 27 \cdot BC \Rightarrow BC = \frac{45^2}{27} = \frac{5 \cdot 15 \cdot 45}{27} = 75 \text{ cm}$$

$$DC = 75 - 27 = 48 \text{ cm}$$



T.C. $\Rightarrow AC^2 = BC \cdot DC$

$AC^2 = 75 \cdot 48$

$AC = \sqrt{25 \cdot 3 \cdot 16 \cdot 3} = 5 \cdot 4 \cdot 3 = 60 \text{ cm.}$

7/71, 7:

ΔABC

$m(\hat{A}) = 90^\circ$

M - mijl (BC)

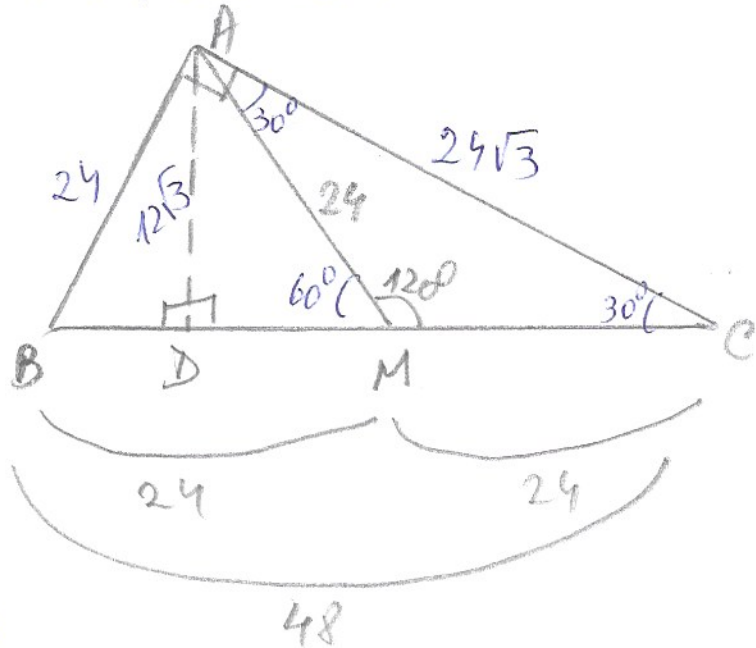
$AD \perp BC$

$DE \parallel BC$

$AM = 24 \text{ cm}$

$m(\hat{AMC}) = 120^\circ$

Cl: $BC, AB, AC, AD = ?$



Sol:

În ΔABC - dr, AM - mediană $\Rightarrow AM = \frac{BC}{2} \Rightarrow BC = 2 \cdot AM \Rightarrow$

$\Rightarrow BC = 2 \cdot 24 = 48 \text{ cm} \Rightarrow BM = MC = AM = \frac{BC}{2} = 24 \text{ cm.}$

În ΔAMC , $AM = MC \Rightarrow \Delta AMC$ - is $\Rightarrow m(\hat{MAC}) = m(\hat{MCA}) = 30^\circ$
 $m(\hat{AMB}) = 180^\circ - m(\hat{AMC}) = 180^\circ - 120^\circ = 60^\circ \Rightarrow \Delta AMB$ - echil.
 ΔAMB - is. \checkmark

În ΔABC , $m(\hat{A}) = 90^\circ$ T.P. $\Rightarrow AC^2 = BC^2 - AB^2$ $\left[AB = 24 \text{ cm} \right]$

$AC^2 = 48^2 - 24^2$

$AC^2 = 24^2 \cdot (2^2 - 1^2) \Rightarrow AC^2 = 24^2 \cdot 3 \Rightarrow AC = \sqrt{24^2 \cdot 3}$

$AD = h_{\Delta dr} = \frac{\text{cat}_1 \cdot \text{cat}_2}{ip} = \frac{AB \cdot AC}{BC} = \frac{24 \cdot 24\sqrt{3}}{48} = 12\sqrt{3} \text{ cm}$

TEMA: cul pag 70/1C, 2b, 3b, 4a, 6.