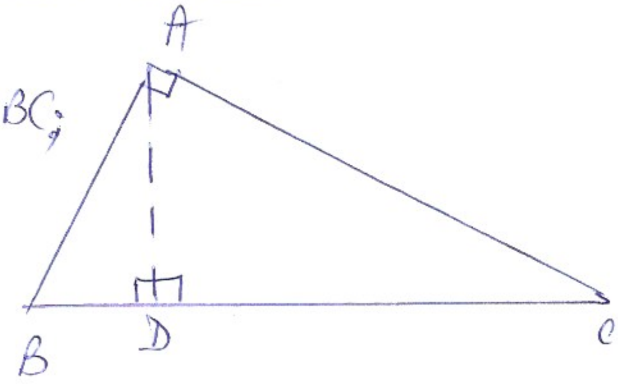
LECȚIA 2 ① 23.04

GEOMETRIE PROF. SCHNAKOVSZKI C.

cls a VII-a A

RELAȚII METRICE ÎN TRIUNGHIUL DREPTUNGHIC – TEOREMA CATETEI



Teor: În ΔABC, $m(\hat{A}) = 90^\circ$, $AD \perp BC$;

TEOREMA ÎNĂLȚIMII:

$$AD^2 = BD \cdot DC$$

TEOREMA CATETEI:

Într-un triunghi dreptunghic, pătratul lungimii unei catete este egal cu produsul dintre lungimea ipotenuzei și proiecția catetei pe ipotenuză.

$$\boxed{\begin{aligned} AB^2 &= BC \cdot BD \\ AC^2 &= BC \cdot CD \end{aligned}}$$

RECIPROCELE TEOREMEI CATETEI:

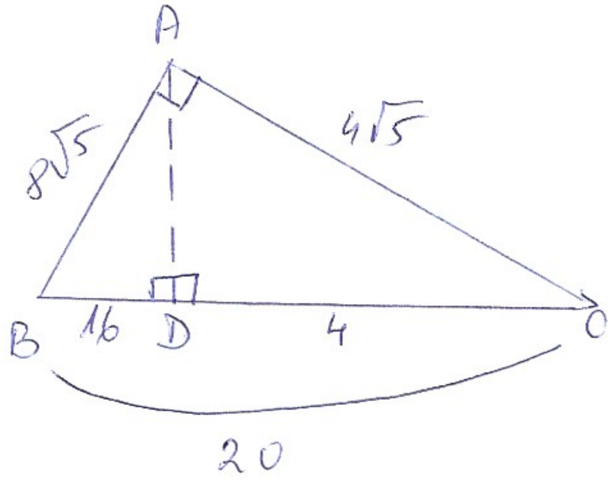
1. Fie ΔABC cu $m(\hat{A}) = 90^\circ$ și $D \in (BC)$. Dacă $AB^2 = BC \cdot BD$, atunci $AD \perp BC$.

2. Fie un ΔABC și $AD \perp BC$, $D \in (BC)$. Dacă $AB^2 = BC \cdot BD$, atunci $m(\widehat{BAC}) = 90^\circ$.

1/61. Ip:

ΔABC, $m(\hat{A})=90^\circ$

$AD \perp BC$, $D \in (BC)$.



a) $BC=20$ cm

$BD=16$ cm

Cl: CD, AB, AC, AD = ?

Dem:

$DC = BC - BD = 20 - 16 = 4$ cm

In ΔABC, $m(\hat{A})=90^\circ \xRightarrow{T.C.} AB^2 = BC \cdot BD$

$AB^2 = 16 \cdot 20$

$AB = \sqrt{16 \cdot 20} = 4\sqrt{4 \cdot 5} = 8\sqrt{5}$ u

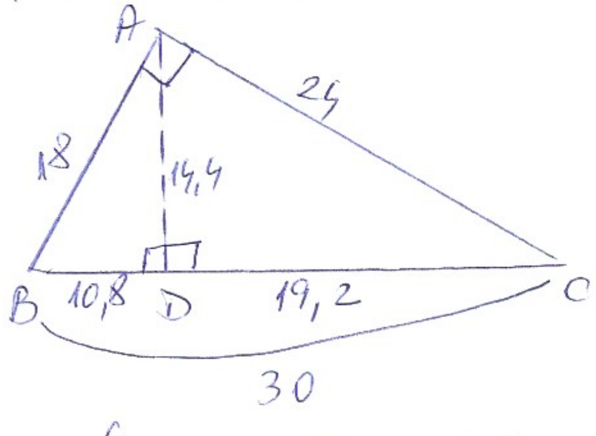
$AC^2 = BC \cdot CD$

$AC = \sqrt{20 \cdot 4} = \sqrt{4 \cdot 5 \cdot 4} = 4\sqrt{5}$ u

$\xRightarrow{T.\hat{I}} AD^2 = BD \cdot DC$

$AD^2 = 16 \cdot 4$

$AD = \sqrt{16 \cdot 4} = 4 \cdot 2 = 8$ cm



b) $AB = 18$ cm

$BC = 30$ cm

BD, CD, AC, AD = ?

Dem:

In ΔABC, $m(\hat{A}) = 90^\circ \xRightarrow{T.C}$

$AB^2 = BC \cdot BD$

$18^2 = 30 \cdot BD \Rightarrow BD = \frac{18^2}{30} = \frac{\cancel{18}^{6} \cdot 18}{\cancel{30}_{10}} = \frac{108}{10} = 10,8$ cm

$DC = BC - BD = 30,0 - 10,8 = 19,2$ cm

$\xRightarrow{T.\hat{I}} AD^2 = BD \cdot DC \Rightarrow AD^2 = 10,8 \cdot 19,2 = \frac{108}{10} \cdot \frac{192}{10} = \frac{36 \cdot 3 \cdot 64 \cdot 3}{10^2}$

$AD = \sqrt{\frac{36 \cdot 64 \cdot 3^2}{10^2}} = \frac{6 \cdot 8 \cdot 3}{10} = \frac{48 \cdot 3}{10} = 14,4$ cm

$\overset{T.C}{\Longrightarrow} AC^2 = DC \cdot BC$

$AC^2 = 19{,}2 \cdot 30 = \frac{192}{1\cancel{0}} \cdot 3\cancel{0} = 576$

$AC = \sqrt{576} = 24$ cm

3/61 Ip:

ΔABC, $m(\hat{A}) = 90^\circ$

AM = mediană

$M \in (BC)$

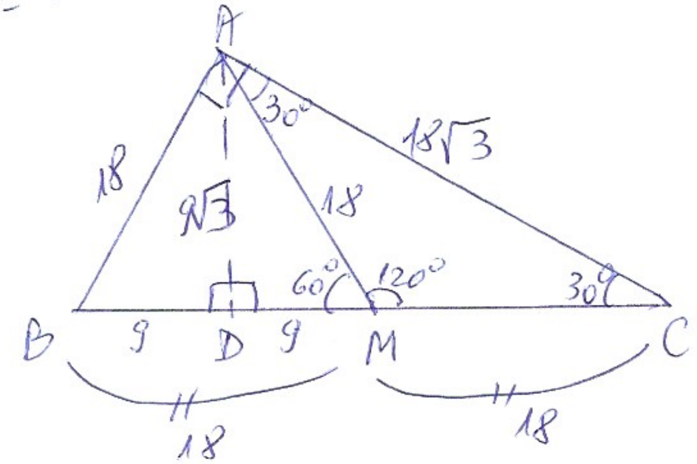
$AM = 18$ cm

$m(\widehat{MAC}) = 30^\circ$

Cl: a) $\frac{BD}{CD} = ?$

$AD \perp BC$

b) $\mathcal{A}, P_{\Delta ABC} = ?$



Dem:

$\left.\begin{array}{l}\text{Din } \Delta ABC \text{ drept.} \\ AM - \text{mediană}\end{array}\right\} \Rightarrow AM = \frac{BC}{2}$

$18 = \frac{BC}{2} \Rightarrow$

$\Rightarrow BC = 18 \cdot 2 = 36$ cm. $\Rightarrow$

$\Rightarrow BM = MC = AM = 18$ cm $\Rightarrow$

$\Rightarrow \Delta AMC$ – is. $\Rightarrow m(\hat{C}) = m(\widehat{CAM}) = 30^\circ \Rightarrow$

$\Rightarrow m(\widehat{AMC}) = 180^\circ - (30^\circ + 30^\circ) = 180^\circ - 60^\circ = 120^\circ \Rightarrow$

$\left.\begin{array}{l}\Rightarrow m(\widehat{AMB}) = 180^\circ - 120^\circ = 60^\circ \\ \Delta AMB - \text{isoscel}\end{array}\right\} \Rightarrow \Delta AMB$ – echilateral $\Rightarrow$

$\Rightarrow \boxed{AB = 18 \text{ cm}}$

$AD = h$ $\Rightarrow$

$\Rightarrow AD$ = mediana $\Rightarrow BD = DM = \frac{18}{2} = 9$ cm $\Rightarrow \frac{BD}{CD} = \frac{9}{27} = \boxed{\frac{1}{3}}$

În ΔABC, $m(\hat{A}) = 90^\circ \overset{T.Î}{\Longrightarrow} AD^2 = BD \cdot DC \Rightarrow AD = \sqrt{9 \cdot 27} = 9\sqrt{3}$ cm

$\overset{T.C}{\Longrightarrow} AC^2 = CD \cdot CB \Rightarrow AC = \sqrt{27 \cdot 36} = \sqrt{9 \cdot 3 \cdot 36}$

$\mathcal{A}_{\Delta ABC} = \frac{c_1 \cdot c_2}{2} = \frac{\overset{9}{\cancel{18}} \cdot 18\sqrt{3}}{\cancel{2}_1} = 162\sqrt{3}$ cm^2 $\quad AC = 3 \cdot 6\sqrt{3} = \boxed{18\sqrt{3}}$

$P_{\Delta ABC} = AB + AC + BC = 18 + 18\sqrt{3} + 36 = 54 + 18\sqrt{3} = 18(3 + \sqrt{3})$ cm.

5/62. Ip:

ABCD - dreptunghi

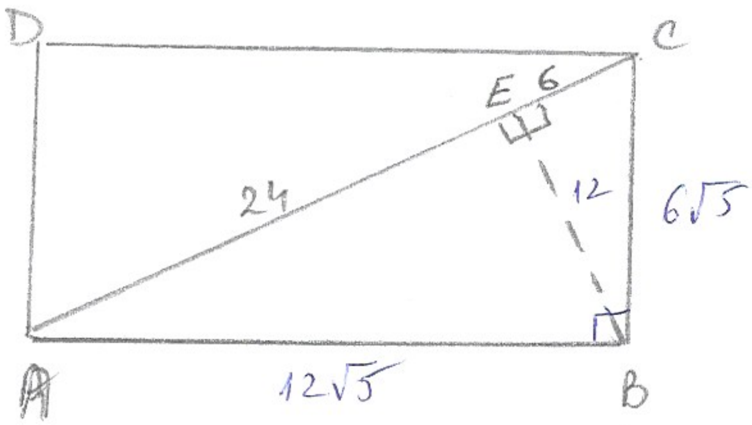
$BE \perp AC$

$E \in (AC)$

$EC = 6$ cm

$AE = 24$ cm

Cl: P, $\mathcal{A}_{ABCD}$ = ?



$AC = AE + EC = 24 + 6 = 30$ cm

Dem:

În $\triangle ABC$, $m(\hat{B}) = 90^\circ \xRightarrow{T.Î.} BE^2 = AE \cdot EC$

$BE^2 = 24 \cdot 6$

$BE = \sqrt{6 \cdot 4 \cdot 6} = 6 \cdot 2 = 12$ cm

$\xRightarrow{T.C.} AB^2 = AC \cdot AE$

$AB^2 = 30 \cdot 24$

$AB = \sqrt{6 \cdot 5 \cdot 6 \cdot 4} = 6 \cdot 2\sqrt{5} = 12\sqrt{5}$ cm

$\xRightarrow{T.C.} BC^2 = CE \cdot CA$

$BC^2 = 6 \cdot 30 \Rightarrow BC = \sqrt{6 \cdot 6 \cdot 5} = 6\sqrt{5}$ cm

$P_{ABCD} = 2 \cdot 12\sqrt{5} + 2 \cdot 6\sqrt{5} = 24\sqrt{5} + 12\sqrt{5} = 36\sqrt{5}$ cm

$\mathcal{A}_{ABCD} = L \cdot l = 12\sqrt{5} \cdot 6\sqrt{5} = 72 \cdot 5 = 360$ cm^2

6/62. Ip:

ABCD - trapez dr.

$m(\hat{A}) = m(\hat{D}) = 90^\circ$

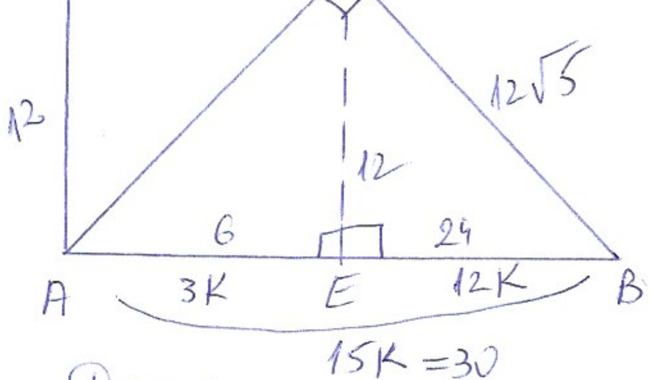
$AB \parallel CD$

$\{CD, AB\}$ d.p. $\{3, 15\}$

$AC \perp BC$

$AD = 12$ cm

Cl: $\mathcal{A}$, P_{ABCD} = ?



Dem:

Fie $CE \perp AB$

Din $\{CD, AB\}$ d.p. $\{3, 15\} \Rightarrow \frac{CD}{3} = \frac{AB}{15} = K \Rightarrow CD = 3K$

$AB = 15K$

$EB = AB - CD = 15K - 3K = 12\,K$

În ΔACB, $m(\hat{C}) = 90^\circ \xRightarrow{T.Î.} CE^2 = AE \cdot EB$

$12^2 = 3K \cdot 12K$

$144 = 36 \cdot K^2 \Rightarrow K^2 = \frac{144}{36} = 4$

$\boxed{K = 2}$

$CD = 3 \cdot 2 = \boxed{6\text{ cm}}$

$AB = 15 \cdot 2 = \boxed{30\text{ cm}} \Rightarrow BE = 30 - 6 = 24\text{ cm}$

$\xRightarrow{T.C.} CB^2 = BA \cdot BE$

$CB^2 = 30 \cdot 24$

$CB = \sqrt{6 \cdot 5 \cdot 6 \cdot 4} = 6 \cdot 2 \cdot \sqrt{5} = \boxed{12\sqrt{5}\text{ cm}}$

$\mathcal{A} = \frac{(B+b) \cdot h}{2} = \frac{(30+6) \cdot \cancel{12}^{6}}{\cancel{2}} = 36 \cdot 6 = 216\text{ cm}^2$

$\mathcal{P} = AB + BC + CD + AD = 30 + 12\sqrt{5} + 6 + 12 = 48 + 12\sqrt{5} =$

$= 12 \cdot (4 + \sqrt{5})\text{ cm}.$

TEMĂ până marți: culegere

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